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VIEW

OF

Sir *ISAAC NEWTON*'s METHOD

For Comparing

The *Reſiſtance* of SOLIDS.

By CHRISTOPHER ROBINSON.

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BRITISH MUSEUM

TO

Sir JACOB ACWORTH, *Knt.*

SURVEYER

OF

His Majeſty's *NAVY.*

SIR,

YOUR *Diſpoſition to promote whatever is ingenious, juſtly demands the grateful Acknowledgment of all Lovers of Art: And as I have my ſelf, in a moſt particular Manner, experienced your Goodneſs, pray Leave to lay before you, this little Treatiſe, on Sir* Iſaac Newton's *Solid of the leaſt Reſiſtance; which, tho' it has been handled by ſeveral able* Mathematicians, *yet, has been looked upon as a Matter of mere Speculation,* *'till you were pleaſed (among your many ſucceſsful Endeavours, to improve the* Royal Navy) *in a more particular Manner to conſider and reduce this to Practice: I am thoroughly ſenſible, to treat on the Works of the* Illuſtrious Author, *is a Task I am no way* equal *to; therefore beg you will paſs by the Meanneſs of the Performance, and accept of the Endeavours of*

Sir,

Your moſt Dutiful and
Moſt Obedient Servant,

C. Robinſon.

London, *April* 5. 1734.

A VIEW of *Sir* ISAAC NEWTON'S *Thirty Fourth Propoſition* of the *Second Book* of his *Mathematical Principles* of NATURAL PHILOSOPHY.

THE Reſiſtance of a Body moving in a Fluid Medium, ariſes partly from the Denſity of the Fluid in which it moves, partly from the Number of the Particles which ſtrike the Body at the ſame Time, and the Angles in which they ſtrike jointly.

The Denſity of the Fluid is not here conſidered: All the Bodies whoſe Reſiſtances are here compared, being ſuppoſed to move in the ſame Fluid; as all in *Air*, *Water*, *Oil*, *&c.* and not One to move in *Air*, a Second in *Water*, a Third in *Oil*, *&c.*

The Number of the Particles of a Fluid which ſtrike on Bodies moving in it, are as the Areas of the Greateſt Tranſverſe Sections of thoſe Bodies.

The Angle in which the Particles of the Fluid ſtrike the Body, is the Angle which the Sides or Surface of the Body make with the Direction of the

Motion, and is called the Angle of Incidence of the Particles of the Fluid.

Thus, if the Parallelogram ADEB, *Fig.* 1. and the Triangle AVB were both to move in the ſame Fluid, the Number of Particles that would ſtrike on thoſe Figures would be equal, becauſe the Figures are contained under equal Baſes; but the Angle of Incidence in which they ſtrike unequal, being Perpendicular or a Right Angle on the Parallelogram, and Oblique or the Acute Angle VBE on the Triangle.

If a Curve Line FGHV, *Fig.* 2. be moved in a Fluid Medium in the Direction of its Axis AV; then ſuppoſing AB, BC infinitely Small or Fluxions of the Abſciſſa: DF, EG Fluxions of the Ordinate: FG, GH Fluxions of the Curve, the Angle of Incidence of the Particles of the Fluid in the Point H will be KHT or CTH, that is, an Angle made by the Interſection of a Tangent applied in the Point H, with the Axis AV produced in T, and is ſimilar to the infinitely Small Triangle EHG.

The Sum of the Reſiſtances on every Point of the Curve is the Reſiſtance of the whole Curve Line, and if the Fluxions of the Ordinates be every where equal, that is, if FD=GE, &c. the Number of the Particles of the Fluid which ſtrike upon every infinitely little Part of the Curve FG, GH, &c. will be equal, and the Reſiſtances of every ſuch infinitely little Part or Fluxion of the Curve will be as GE ſquared, divided by GH ſquared, or the Square of the Fluxion of the Ordinate divided by the Square of the Fluxion of the Curve, or ſuch a Proportion of the Homologous Sides of any Similar Triangle.

But, if a Solid generated by revolving the ſame Curve FGHV about an Axis be moved in a Fluid Medium in the Direction of its Axis: The Reſiſtance on every ſuch infinitely little Part of the Solid, generated by the infinitely little Triangles FGD, GHE will not be in the ſame Proportion as on the Curve Line: For the Fluxions of the Ordinates being equal, the Number of the Particles that ſtrike on every of the Elementa or Fluxions of the Curve, conſidered as a Line is equal, but conſidered as a Solid unequal (tho' the Angle of Incidence is every where the ſame) becauſe the Surface generated by DF at the Diſtance GB or DA is greater than the Surface generated by revolving EG at the Diſtance EB or HC: the Whole of which is contained in Sir *Iſaac Newton's*

PRINCIP. MATHEM. *Lib.* II. *Prop.* 34. *Theor.* 28.

" If a Globe and Cylinder of equal Diameters be
" moved with equal Velocity, in a Rare or Thin Me-
" dium, conſiſting of Particles equal and equally di-
" ſtant from one another, in the Direction of the
" Axis of the Cylinder: the Reſiſtance of the Globe
" would be half as much as the Reſiſtance of the
" Cylinder.

" For, becauſe the Action of the Medium is the
" Same on the Body (by *Cor.* 5. *of the Laws*) whe-
" ther the Body be moved in a Medium at Reſt, or
" the Particles of the Medium, ſtrike the Body at
" Reſt with the ſame Velocity: Let us conſider the
" Body at Reſt, and ſee with what Force it will be
" preſſed by a moving Medium.

" Let therefore ABKI, *Fig.* 3. repreſent a Sphe-
" rical Body deſcribed on the Center C, with the
" Radius CA, and let the Particles of the Medium
" fall with the given Velocity upon that Spherical
" Body, in Right Lines parallel to AC: Let FB
" be a Right Line of this Kind, in it let LB be ta-
" ken equal to the Semidiameter CB, and let BD
" be drawn touching the Sphere in B, on KC and

“ BD let fall the Perpendiculars BE, DL, and the Force with which a Particle of the Medium falling Obliquely in the Right Line FB, ſtrikes the Sphere in B, will be to the Force with which the ſame Particle would ſtrike the Cylinder ONGQ circumſcribed to the Sphere, with the ſame Axis ACI in *b*, as LD to LB, or BE to BC. Again, the Effect of this Force to move the Globe in the Direction of its Incidence FB or AC is to the Power to move the ſame in the Direction of its own Determination, that is, in the Direction of the Right Line BC in which it ſtrikes the Globe Perpendicularly, as BE to BC; and by Compounding theſe Ratio's, the Force of the Particle falling Obliquely on the Globe in the Right Line FB, to move the ſame in the Direction of its Incidence, is to the Power of the ſame Particle falling perpendicularly on the Cylinder in the ſame Right Line, as BE ſquared to BC ſquared; wherefore, if a Perpendicular *b*HE be erected to the Circular Baſe of the Cylinder NAO, and *b*E be equal to the Radius AC, and *b*H equal to $\frac{\text{BE ſquared}}{\text{BC}}$, *b*H will be to *b*E as the Effect of the Particle on the Globe to the Effect of the Particle on the Cylinder, and therefore, the Solid which is made up of an infinite Number of Right Lines here repreſented by *b*H, will be to the Solid which is made up of an infinite Number of Right Lines here repreſented by *b*E, as the Effect of all the Particles on the Globe to the Effect of all the Particles on the Cylinder. But the former Solid is a Paraboloid, deſcribed through the Vertex C on the Axis CA and Parameter CA, and the latter Solid is a Cylinder circumſcribed to the Paraboloid: And it is known, that the Paraboloid is half the circumſcribed Cylinder. Therefore, the

" whole Force of the Medium on the Globe is half " the Force of the ſame on the Cylinder. And " therefore, if the Particles of the Medium be at " Reſt, and the Globe and Cylinder be moved with " an equal Velocity, the Reſiſtance of the Globe " would be half the Reſiſtance of the Cylinder. " Q. E. D.

SCHOLIUM.

" By the ſame Method the Reſiſtance of other " Figures may be compared, and ſuch may be found, " which are beſt adapted to continue their Motions " in Reſiſting Mediums. As if on the Circular Baſe " CEBH, *Fig*. 4. which is deſcribed on the Center " O, the Fruſtum of a Cone is to be conſtructed, " which meets with the leaſt Reſiſtance of all Fruſtums " of the ſame Baſe and Altitude, and moving " in the Direction of its Axis towards D: Biſect " the Altitude OD in Q, and produce OQ to S, " that QS may be equal to QC, and S will be the " Vertex of the Cone whoſe Fruſtum it ſought.

" Hence, Seeing the Angle CSB is always Acute, " it follows, that if a Solid ADBE, *Fig*. 5. be generated " by the Convolution of an Elliptical or " Oval-like Figure ADBE about its Axis AB, and " the generating Figure be touched by three Right " Lines FG, GH, HI, in the Points F, B and I, " in ſuch a Manner that GH be perpendicular to " the Axis in the Point of Contact B, and HI, FG " with the ſaid Line GH contain the Angles FGB, " BHI, 135 Degrees; the Solid which is generated " by turning this Figure ADFGHIE about its Axis, " CB, will be leſs reſiſted than the former Solid, if " both move in the Direction of the Axis AB with " the End B foremoſt: which Propoſition, I conceive, " may be uſeful in *Ship-Building*.

" But if the Figure DNFG be a Curve of ſuch a
" Kind: If from any Point N, a Perpendicular NM
" be let fall on the Axis AB, and from a given
" Point G be drawn a Right Line GR Parallel to a
" Right Line, touching the Figure in N, and cut-
" ting the Axis produced in R; MN will be to GR,
" as GR cubed to 4BR×GB ſquared: The Solid
" which is formed by revolving this Figure about
" its Axis AB, will meet with leſs Reſiſtance, in
" moving in the aforeſaid rare Medium from A to-
" ward B than any other Circular Solid deſcribed
" with the ſame Length and Breadth".

The Method here given by Sir *Iſaac Newton*, extends to the Inveſtigating the Reſiſtances of Solids in general, whether they be generated by revolving the generating Lines about an Axis, or by any other known Law, that is, whether the Baſes and all the Tranſverſe Sections be Circles, Polygons, *&c.*

If the Tranverſe Sections be all Right-angled Parallelograms, and the reſpective Sides of all thoſe Parallelograms, parallel to one another, as in a Wedg or Figure of a Wedg-like Form, whoſe Sides, inſtead of being ſtraight, are Curved: The Reſiſtances of ſuch Solids will be, the Reſiſtances of the generating Lines multiplied into the Depth.

In Figures of this Kind, the Plains or flat Sides may be ſuppoſed to move Horizontally; however, tho this may be of Uſe in aſſiſting the Imagination, it is not any ways neceſſary, that they ſhould be confined to ſuch a Motion, but may be at Liberty to move any how, ſo it be in the Direction of the Axis of the generating Lines, and plain Superficies.

PROP. I.

To Inveſtigate the Reſiſtances *of* Solids *contained under* Straight Lines.

LET the Parallelogram ABCD, *Fig.* 6. be the Side of a Parallelopipedon, ſuppoſed to move Horizontally in the Direction of FK, whoſe Altitude AB is denoted by h, the half Breadth BK by b, and the Depth by d.

Caſe 1. If the aboveſaid Parallelopipedon be Cyphered away from Bottom to Top, in the Right Line EB, *Fig.* 7. the Reſiſtance of the Parallelopipedon will be to the Reſiſtance of this Solid, as $hh+dd$ to dd, or putting $2hbd$, or the Content of the Parallelopipedon for its Reſiſtance; the Reſiſtance of this Solid will be $\frac{2hbd^3}{hh+dd}$, for, by the aforecited Propoſition $EBq . AEq :: AB$ to $\frac{AB \times AEq}{EBq}$ $= BL$, whence the Reſiſtance of this Solid will be to that of the former, as the Line BL is to the Line AB, or as the Area of the Rectangle BLNO is to the Area of the Rectangle ABOE, which (becauſe the Slant Side EB is every where of the ſame Breadth) is the ſame Thing, as if the Parallelopipedons were compared with one another.

Caſe 2. If the Horizontal Plane be cyphered away to the Triangle AKD, *Fig.* 8. and not from Bottom to Top, as in the firſt Caſe, the Reſiſtance of this Solid will be $\frac{2hb^3d}{hh+bb}$, for, as before, AKq.

$AHq :: AB. \frac{AB \times AHq}{AKq} = BL$, or (for the Reaſon given in the former *Caſe*) the Reſiſtance of this Solid will be to the Reſiſtance of its circumſcribed Parallelopipedon, as BL to BA.

Caſe 3. Let now the Parallelopipedon be cyphered away both Ways together, and the Reſiſtance of this Piramidal Figure will be $\frac{hbd^3}{hh+dd} + \frac{hdb^3}{hh+bb}$ or half the Sum of the Reſiſtances of the Solids in the former *Caſes:* For, the Particles of the Fluid ſtrike this Body with the ſame Obliquity as the former, on all its Sides reſpectively, and the Number of Particles that ſtrike this Solid, will be half ſo many, as ſtrike both the other Solids on their reſpective Sides at the ſame Time; or, as was obſerved before, the Baſes of theſe Solids being equal, the Number of the Particles of the Fluid that ſtrike at the ſame Time will alſo be equal.

If the Depth and half Breadth of the foregoing Solids be the ſame, or $b=d$ the Reſiſtances of all thoſe Solids will be equal. For

1ſt, The Reſiſtance of the Solid in the firſt *Caſe* being $\frac{2hd^3b}{hh+dd}$ it is plain, that by ſubſtituting b for d and d for b (which, becauſe $b=d$, does not alter the Value of the Expreſſion) $\frac{2hbd^3}{hh+dd}$ will become $\frac{2hdb^3}{hh+bb}$ the Reſiſtance, as in the ſecond *Caſe*.

2. By Subſtituting as before *b* for *d*, and *d* for *b*, the Value of $\frac{hbd^3}{hh+dd}+\frac{hdb^3}{hh+bb}$ will become $\frac{2hdb^3}{hh+bb}$ or $\frac{2hbd^3}{hh+dd}$ as before, or may in this *Caſe* be expreſſed by $\frac{2hb^4}{hh+bb}=\frac{2hd^4}{hh+dd}$.

Now, if the Altitude AB or $h=66,9$, and $b=d=33\frac{1}{3}$, then BL $=13,3053$ very near: The Solid Content of the Parallelopipedon will be 148666, and if that be put for its Reſiſtance, the Reſiſtance of the before-mentioned Solids will be 29567 nearly.

The Content of the Solids in the Firſt and Second *Caſes* is half the circumſcribed Parallelopipedon, or 74333, by which if the Reſiſtance be divided, there will come forth ,39776.

The Content of the Piramid or Solid in the Third *Caſe* is one Third of the Parallelopipedon or 49555, by which if the Reſiſtance be divided, there will come forth ,59665: So that the Solids in the Firſt and Second *Caſes* have a greater Solid Content in Proportion to their Reſiſtances than that in the Third *Caſe*.

Suppoſe now the Breadth to be encreaſed and the Depth leſſened, ſo that the Area of the Baſe, and conſequently their Solidities, remain the ſame as before, and the Breadth be in Proportion to the Depth as 3 to 1.

Put the new Half Breadth $=a$, then $3 . 1 :: 2a . \frac{2a}{3}$ the new Baſe: Becauſe, the Depth of the former Parallelopipedon was $2bb$, and that of the Baſe here ſpoken of is $\frac{4aa}{3}$ therefore $\frac{4aa}{3}=2bb$ and multiplying by 3, $4aa=6bb$, $2aa=3bb$, or $aa=\frac{3bb}{2}$ and by Evolution $a=\sqrt{\frac{3bb}{2}}$.

But if it were required to find the Depth, put the Depth $=e$, and then $1 . 3 :: e . 3e$ the new Breadth, and conſequently, $3ee=2bb$, whence $ee=\frac{2}{3}bb$, and by Evolution $e=\sqrt{\frac{2}{3}bb}=b\sqrt{\frac{2}{3}}$.

Let this Parallelopipedon be cyphered away from Bottom to Top, as in the Firſt *Caſe*, and its Reſiſtance will be ſound 21111, which divided by the Solid Content gives ,284005.

If the Parallelopipedon be cyphered off as in the Second *Caſe*, the Reſiſtance will then be 40339, and the Reſiſtance divided by the Solidity ,54268.

If it be cyphered off both Ways, as in the Third *Caſe*, the Reſiſtance will be (as was ſhewn before) half the Sum of the Reſiſtances of the two foregoing Solids or 30725, and the Reſiſtance divided by the Content is ,62C02.

It may be obſerved from what has been ſaid, that the Reſiſtance is very much leſſened when the Parallelopipedon is cyphered off as in the Firſt *Caſe*, and greatly encreaſed in the Second, from what it was, when the Half Breadth and Depth were equal: Let us now conſider what Proportion the Breadth ſhould bear to the Depth, to make the Reſiſtance, the leaſt poſſible.

Put as before the Baſe $=2bb=2dd$, and let y denote the new half Breadth: Then $y . b :: b . \frac{bb}{y}$ the new Depth, and the Reſiſtance of this Parallelopipedon, when it is cyphered away as in the Firſt *Caſe* from Bottom to Top, may be expreſſed by

$\frac{2hbb\times\frac{b^4}{yy}}{hh+\frac{b^4}{yy}}$, now $2hbb\times\frac{b^4}{yy}=\frac{2hb^6}{yy}$ and $hh+\frac{b^4}{yy}=$

$\frac{h^2y^2+b^4}{yy}$, whence $\frac{2hbb\times\frac{b^4}{yy}}{hh+\frac{b^4}{yy}}=\frac{2hb^6}{h^2y^2+b^4}$ the Fluxion of which is $-\frac{4h_3b^6y\dot{y}}{\overline{h^2y^2+b^4}|^2}$ which if a Minimum,

then $-\frac{4h^3b^6y\dot{y}}{\overline{h^2y^2+b^4}|^2}=0$, and multiplying by

$\overline{h^2y^2+b^4}|^2$; $-4h^3b\ y\dot{y}=0$, divided by $\dot{y}$, $-4h^3b^6y=0$, and conſequently, $-y=0$, that is, if the Altitude and Area of the Baſe remain the ſame, the Reſiſtance will be continually leſſened by encreaſing the Breadth and leſſening the Depth, till the Depth vaniſhes.

In the Second *Caſe* the Reſiſtance may be expreſſed by $\frac{2hb^2y^2}{h^2+y^2}$ the Fluxion of which is

$\frac{4h^3b^2y\dot{y}+4hb^2y^3\dot{y}-4hb^2y^3\dot{y}}{\overline{hh+yy}|^2}=\frac{4h^3b^2y\dot{y}}{\overline{hh+yy}|^2}$: if this be a Minimum, then $4h^3b^2y=0$, and conſequently $y=0$, that is, the Reſiſtance will keep in a continual Decreaſe as the Breadth decreaſes.

In the Third *Caſe*, the Reſiſtance may be expreſſed by $\frac{hb^6}{h^2y^2+b^4}+\frac{hb^2y^2}{h^2+y^2}$ or inſtead thereof we may uſe $\frac{b^4}{h^2y^2+b^4}+\frac{y^2}{h^2+y^2}$ the Fluxion of which if a Minimum, is $\frac{2h^2y\dot{y}}{h^4+2h^2y^2+y^4}-\frac{2h^2b^4y\dot{y}}{h^4y^4+2h^2y^2+b^8}=0$, and dividing both Sides by $2h^2y\dot{y}$ we ſhall have $\frac{1}{h^4+2h^2y^2+y^4}=\frac{b^4}{h^4y^4+2h^2y^2b^4+b^8}$, or $h^4y^4+2h^2y^2b^4+b^8=b^4h^4+2h^2b^4y^2+b^4y^4$, and taking away $2h^2y^2b^4$ from both Sides of the Equation we ſhall have $\frac{b^8-b^4h^4}{b^4-h^4}=y^4$, or $b^4=y^4$, and by Evolution $b=y$, that is, the Reſiſtance of this Piramidal Figure will be the leaſt when the half Breadth and Depth are equal.

COROLLARY.

The Reſiſtance of a right Cone is to the Reſiſtance of its Circumſcribed Piramid as the Solidity of the Cone is to the Solidity of the Piramid, and as the Surface of the Cone to the Surface of the Piramid. For, 1ſt, The Angle of Incidence of the Particles of the Fluid is the ſame on both Solids.

2d, The Curve Surface of a right Cone is equal to a Triangle whoſe Baſe is the Periphery of the Baſe of the Cone, and its Altitude equal to the Side of the Cone.

3d, The Surface of a right Piramid is equal to a Triangle, whoſe Baſe is the Perimeter of the Baſe of the Piramid, and its Altitude equal to the Perpendicular of the Iſoſceles Triangles that compoſe the ſame: Therefore, the Number of the Particles of the Fluid, that ſtrike both the Solids at the ſame Time and with the ſame Angle of Incidence, are as their Surfaces.

4th, The Solidity of a right Piramid is One Third of its circumſcribed Parallelopipedon, and the Solidity of a right Cone is One Third of its circumſcribed Cylinder, therefore their Solidities are as their Baſes; and the Area of a Square is to the Area of its inſcribed Circle, as the Perimeter of the Square to the Periphery of the Circle: For, let r be the Radius of a Circle, $2r$ the Diameter, or Side of the circumſcribed Square, and $8r$ the Perimeter, then if c be put for the Circumference of the Circle, it will be $8r \,.\, c :: 4rr \,.\, \frac{rc}{2}$ the known Area of the Circle.

PROP.

PROP. II.

The Altitude of a Piramid *being given, to determine the* Baſe, *when the* Reſiſtance *ſhall be the leaſt that can be in Proportion to the* Solidity.

PUT the Altitude VD$=h$, *Fig.* 9. and ED, or half the Side of the Baſe, $=y$.

The Reſiſtance of the Piramid is $\frac{hyy}{yy+hh} \times 4yy$, as was ſhewn before, or $\frac{4hy^4}{yy+hh}$: the Solid Content is, $\frac{4hyy}{3}$, by which, if the Reſiſtance be divided we ſhall have $\frac{12hy^4}{4h^3y^2+4hy^4} = \frac{3y^2}{h^2+y^2}$ the Fluxion of which is $\frac{6h^2y\dot{y}+6y^3\dot{y}-6y^3\dot{y}}{\overline{h^2+y^2}|^2}$ which if a Minimum, then $6h^2y\dot{y}=0$, and dividing by $6h^2\dot{y}$, $y=0$, that is, the Baſe will continually decreaſe till it vaniſhes.

If the Content be divided by the Reſiſtance, it will be $\frac{4h^3y^2+4hy^4}{12hy^4} = \frac{h^2+y^2}{3y^2}$ the Fluxion of which is $\frac{6y^3\dot{y}-6h^2y\dot{y}-6y^3\dot{y}}{9y^4} = -\frac{2h^2y\dot{y}}{3y^4}$, if this be a Minimum, then $-2b^2yy=0$, and dividing by $2b^2y$, we ſhall have $-y=0$, that is, ED will increaſe in Infinitum, to make the Quotient of the Solid Content, divided by the Reſiſtance the leaſt that can be.

And what has been ſaid of the Piramid, may be applied to the Cone, or any other Piramidal Solid whoſe tranſverſe Sections are regular Polygons.

PROP. III.

To Inveſtigate the Reſiſtance of a Paraboloid.

LET ENB, *Fig.* 10. be the Parabola given, draw FK parallel to the Axis QB, and let fall the Perpendicular NM from any Point of the Curve on the Axis QB, and draw NI Perpendicular to the Curve in the Point N, then by the aforecited Propoſition of Sir *Iſaac Newton*, NI*q* . IM*q* : : QB . HF, and if this be done in every Point of the Parabola, the Point H will by its Motion deſcribe the Curve QHD, and the Reſiſtance of the Paraboloid generated by revolving the Parabola QENB about its Axis QB will be to the Reſiſtance of the Cylinder generated by the Parallelogram QELB in the ſame Rotation, as the Content of the Solid generated by revolving the Figure BQHDLB, about the ſame Axis QB, is to the Content of the Circumſcribed Cylinder. It remains then to Inveſtigate the Content of the aforeſaid Solid: in order to which,

Put the Altitude $QB=b$, the Ordinate $NM=y$, the Subnormal IM which in this Curve is a conſtant Quantity $=a$, equal to half the Parameter: For, apply the Ordinate nm, infinitely near to NM; the Infinitely little Triangle rNn is ſimilar to the Triangle

angle MNI, therefore $rN \,.\, MN :: rn \,.\, IM$, or the Subnormal $IM = \frac{MN \times rn}{rN} = \frac{y\dot{y}}{\dot{x}}$, but in the Parabola, putting p for the Parameter, $px = yy$ and $p\dot{x} = 2y\dot{y}$ therefore $\frac{y\dot{y}}{\dot{x}} = \frac{p\dot{x}}{2\dot{x}} = \frac{1}{2}p$.

By Subſtituting h for QB, y for NM and a for IM: inſtead of $NIq \,.\, IMq :: QB \,.\, HF$, we ſhall have $aa+yy \,.\, aa :: h \,.\, \frac{aah}{aa+yy}$ the Value of HF, according to the Property of the Parabola.

Now, if c ſtands for the Circumference of a Circle in general, and r for its Radius, then $4rr \,.\, \frac{rc}{2} ::$ $4yy \,.\, \frac{cyy}{2r}$ or $\frac{c}{2r} \times yy$ will be the Area of any particular Circle whoſe Radius is y.

The Baſe therefore of this Solid is $\frac{c}{2r} \times yy$ its Fluxion is $\frac{c}{2r} \times 2y\dot{y}$ the Fluxion of the Baſe, which multiplied into $\frac{aah}{aa+yy}$ gives $\frac{c}{2r} \times aah \times \frac{2y\dot{y}}{aa+yy}$ the Fluxion of the Solid.

In Order to find the Fluent of which, put $aa+yy=z$, then $2y\dot{y}=\dot{z}$ and by Subſtituting $\dot{z}$ for $2y\dot{y}$ the Fluxion of the Solid will be $\frac{c}{2r}\times aab\times\frac{\dot{z}}{z}$: and becauſe the Fluxion of any Logarithm is equal to the Fluxion of the abſolute Number divided by the abſolute Number: therefore the Fluent of $\frac{c}{2r}\times aab\times\frac{\dot{z}}{z}$ will be $\frac{c}{2r}\times aab\times$ Log. z, and reſtoring $aa+yy$ for z, it will be $\frac{c}{2r}\times aab\times$ Log. $\overline{aa+yy}$ but becauſe Log. $\overline{aa+yy}$ muſt vaniſh when $y=0$, therefore the true flowing Quantity or Content of the Solid enquired after will be $\frac{c}{2r}\times aab\times$ Log. $\frac{\overline{aa+yy}}{aa}$, the Reſiſtance of the Paraboloid required.

Put as before $b=66,9$ and $y=33\frac{1}{3}$, then will aa be 68,9608: The Content of the circumſcribed Cylinder will be 233525, and that of the Paraboloid 116762, for the Paraboloid is known to be half the circumſcribed Cylinder.

If the Reſiſtance of the Cylinder be put equal to its Solid Content, the Reſiſtance of the Paraboloid, will by the foregoing Theorem be found equal to 41169, which divided by the Solidity 116762 gives ,35259.

PROP. IV.

To Inveſtigate the Reſiſtance of a Solid *generated by revolving a* Parabola *about an* Ordinate.

LET VBD, *Fig.* 11. be the generating Curve, V the Vertex, AV the Axis, VE an Abſciſſa, and ED an Ordinate of the Parabola applied in the Point E, DK a Perpendicular to the Curve applied in the Point D, and let AB be the Axis of Rotation, in the Direction of which the Solid is to move from A towards B.

Put $VA = r$, $VE = x$, and the Subnormal EK (which in the foregoing Propoſition was ſhewn to be always equal to the Semiparameter) $= a$.

Then $DKq \,.\, EDq :: AB \,.\, FG$ for the Triangle MDC is ſimilar to the Triangle DKE.

But, by the Property of the Parabola $DEq = 2ax$, and $KDq = DEq + EKq = 2ax + aa$, therefore $aa + 2ax \,.\, 2ax :: b \,.\, \frac{2bx}{a+2x} = FG$, and if this be done in every Point of the Curve, the Point H will deſcribe the Curve HFL; and the Reſiſtance of the given Solid will be to the Reſiſtance of its circumſcribed Cylinder, as the Content of the Solid generated by revolving the Curve HFL about the Axis HB to the Content of the circumſcribed Cylinder: In Order to find the Content of the Solid generated by revolving HFL about its Axis HB: Let c be a Standing Expreſſion ſhewing the Proportion of the Area of a Circle to that of its circumſcribed Square. Now $AE = r - x$, therefore the Baſe of the Solid will be $4c \times \overline{rr - 2rx + xx}$, the Fluxion of which is

$4c \times \overline{2x\dot{x}-2r\dot{x}}$ the Fluxion of the Baſe, which multiplied into the Altitude FG$=\frac{2bx}{a+2x}$ gives

$$4c \times \frac{\overline{4bx^2\dot{x}-4brx\dot{x}}}{a+2x} = 4c \times \frac{4bx^2\dot{x}}{a+2x} - 4c \times \frac{4brx\dot{x}}{a+2x}$$ the Fluxion of the Solid.

To find the Fluent of $\frac{4bx^2\dot{x}}{a+2x}$ put $a+2x=z$, then

$z=\frac{z-a}{2}$, $4xx=zz-2za+aa$ and $\dot{x}=\frac{\dot{z}}{2}$, whence

$\frac{4bx^2\dot{x}}{a+2x} = \frac{bz\dot{z}}{2} - ba\dot{z} + \frac{baa\dot{z}}{2z}$ the flowing Quantity of which is $\frac{bz^2}{4} - baz + \frac{baa}{2} \times \text{Log. } z$, and reſtoring $a+2x$ for z, it will be $\frac{b}{4} \times \overline{a-2x}|^2 - ba \times \overline{a+2x} + \frac{baa}{2} \times \text{Log.} \frac{\overline{a+2x}}{a}$.

And to find the Fluent of $-\frac{4brx\dot{x}}{a+2x}$, put as before $a+2x=z$, then $x=\frac{z-a}{2}$ and $xx=\frac{zz-2za+aa}{4}$, whence $2x\dot{x}=\frac{zz-\dot{z}a}{2}$ and $4x\dot{x}=z\dot{z}-\dot{z}a$, by which Subſtitution, the Term $\frac{4brx\dot{x}}{a+2x}$ will become $-br\dot{z}+\frac{abr\dot{z}}{z}$ the Fluent of which is $-brz+abr\times\text{Log.}\,z$, and reſtoring $a+2x$ for z we ſhall have $-br\times\overline{a+2x}+abr\times\text{Log.}\,\frac{\overline{a+2x}}{a}$.

The Aggregate of all which is $\frac{b}{4}\times\overline{a+2x}|^{2}-ba\times\overline{a+2x}-br\times\overline{a+2x}+\overline{\frac{baa}{2}+abr}\times\text{Log.}\,\frac{\overline{a+2x}}{a}$ or $\frac{baa}{4}+abx+bxx-aab-2abx-abr-2brx+\overline{\frac{baa}{2}+abr}\times\text{Log.}\,\frac{\overline{a+2x}}{a}:=-\frac{3aab}{4}-abx+bxx-2brx-abr+\overline{\frac{baa}{2}+abr}\times\text{Log.}\,\frac{\overline{a+2x}}{a}$.

If $x=0$, we ſhall have $-\frac{3aab}{4}-abr$, but if

$x=r$, we ſhall then have $-\frac{3aab}{4}-2abr-brr+\overline{\frac{baa}{2}+abr}\times\text{Log.}\,\overline{\frac{a+2r}{a}}$ which ſubſtracted from $-\frac{3aab}{4}-abr$ leaves the true Fluent $abr+brr-\overline{\frac{baa}{2}-abr}\times\text{Log.}\,\overline{\frac{a+2r}{a}}$, whence the Content of the Solid enquired after, or Reſiſtance of the given Solid is $4c\times\overline{\overline{abr+brr}-\overline{\frac{baa}{2}-abr}\times\text{Log.}\,\overline{\frac{a+2r}{a}}}$

If, as before $b=66{,}9$, $r=33\frac{1}{3}$, then $a=67{,}1341$, and if the Content of the circumſcribed Cylinder be put for its Reſiſtance, the Reſiſtance of the Solid here ſpoken of will be 55342.

PROP. V.

To Inveſtigate the Content of a Solid *generated by revolving the* Parabola VDB *about an* Ordinate AB.

PUT as before, *Fig.* 12. $AB=b$, $VA=r$, $VE=x$, and let p ſtand for the Parameter, then $rp=bb$ and $p=\frac{bb}{r}$ by the Property of the Parabola, and

ED

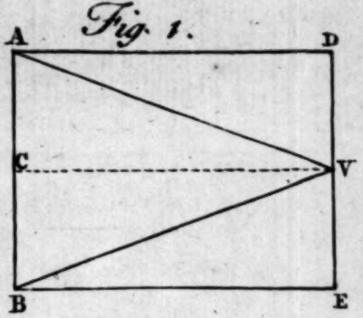
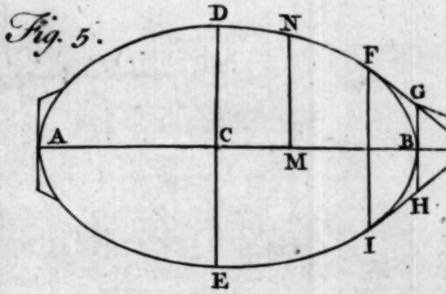


Fig. 1.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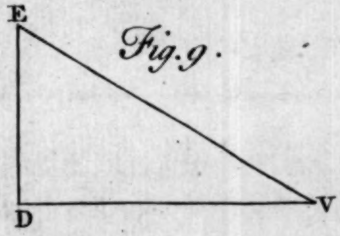


Fig. 9.

Fig. 2.

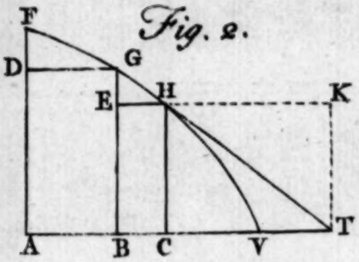
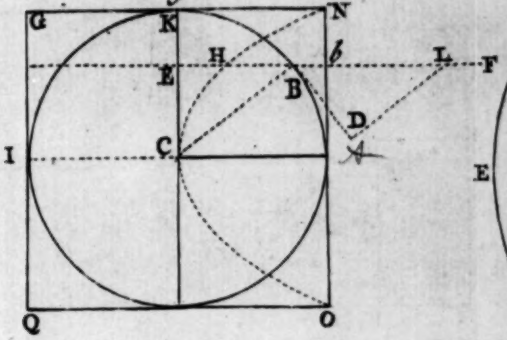


Fig. 3.



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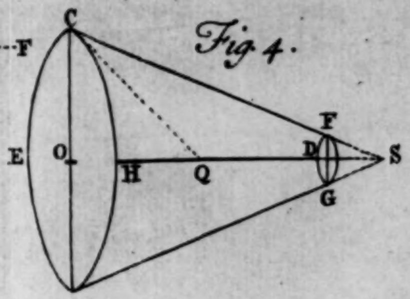

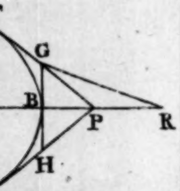

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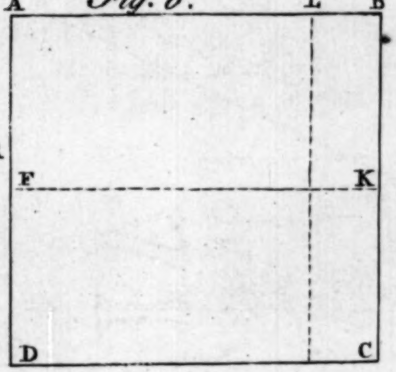


Fig. 7.

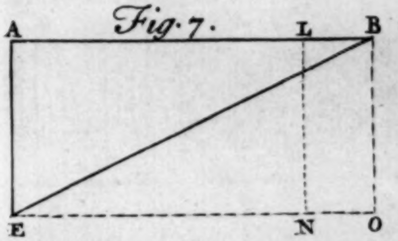


Fig. 8.

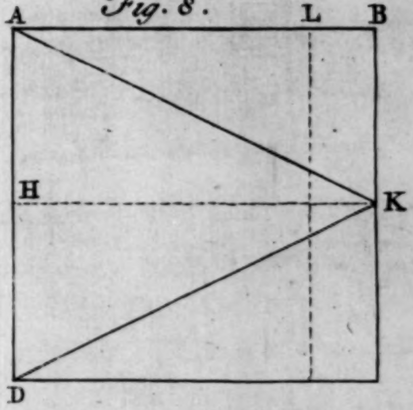


Fig. 11.

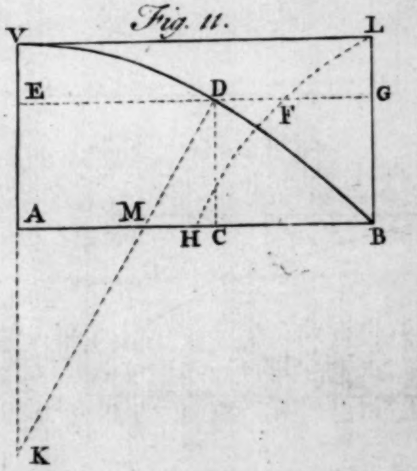


Fig. 10.

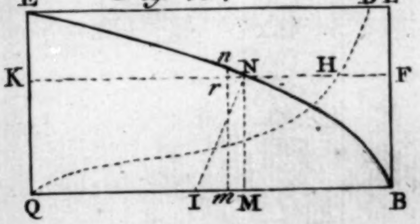
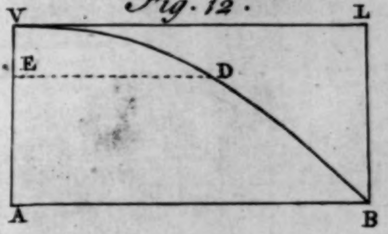


Fig. 12.



J. Mynde sc.

ED $= \frac{bx^{\frac{1}{2}}}{r^{\frac{1}{2}}}$ the Ordinate or Altitude: Put as before $2x\dot{x} - 2r\dot{x}$ for the Fluxion of the Baſe which mutiplied into $\frac{bx^{\frac{1}{2}}}{r^{\frac{1}{2}}}$ the Altitude gives $\frac{2bx^{\frac{3}{2}}\dot{x}}{r^{\frac{1}{2}}} - \frac{2brx^{\frac{1}{2}}\dot{x}}{r^{\frac{1}{2}}}$ the Fluent of which is $\frac{\frac{4}{5}bx^{\frac{5}{2}}}{r^{\frac{1}{2}}} - \frac{\frac{4}{3}brx^{\frac{3}{2}}}{r^{\frac{1}{2}}}$ If $x=0$, the whole will vaniſh, but if $x=r$, we ſhall have $\frac{\frac{4}{5}br^{\frac{5}{2}}}{r^{\frac{1}{2}}} - \frac{\frac{4}{3}br^{\frac{5}{2}}}{r^{\frac{1}{2}}} = \frac{4}{5}br^2 - \frac{4}{3}br^2 = -\frac{8}{15}br^2$ which deducted from 0, gives $\frac{8}{15}br^2$; but br^2 will in this Caſe ſtand for the circumſcribed Cylinder, therefore the Content of the Solid enquired after will be $\frac{8}{15}$ of the Circumſcribed Cylinder: and in Proportion to the Paraboloid as 16 to 15.

If this Solid be of the aforeſaid Dimenſions, its Solidity will be 124546, and its Reſiſtance divided by the Solidity ,44443.

PROP.

PROP. VI.

To Inveſtigate the Reſiſtance of a Spheroid.

PUT the Altitude QB=h, *Fig.* 13. the Abſciſſa MB=x, and the Ordinate NM=y.

Let p ſtand for the Semiparameter, then by the Property of the Ellipſis $yy = \frac{p}{h} \times \overline{2hx-xx}$, and by Evolution $y = \frac{p^{\frac{1}{2}}}{h^{\frac{1}{2}}} \times \overline{2hx-xx}|^{\frac{1}{2}}$ the Fluxion of which is $\dot{y} = \frac{p^{\frac{1}{2}}}{h^{\frac{1}{2}}} \times \frac{h\dot{x}-x\dot{x}}{\overline{2hx-xx}|^{\frac{1}{2}}}$, then $nr . rN :: NM . MT$, Or $\dot{y} . \dot{x} :: y . \frac{\dot{x}y}{\dot{y}}$ the Subtangent, and by Subſtituting the Values of $\dot{y}$, $\dot{x}$ and y, we ſhall have $\frac{p^{\frac{1}{2}}}{h^{\frac{1}{2}}} \times \frac{h\dot{x}-x\dot{x}}{\overline{2hx-xx}|^{\frac{1}{2}}} . \dot{x} :: \frac{p^{\frac{1}{2}}}{h^{\frac{1}{2}}} \times \overline{2hx-xx}|^{\frac{1}{2}} . \frac{2hx\dot{x}-x^{2}\dot{x}}{h\dot{x}-x\dot{x}} = \frac{2hx-xx}{h-x}$ the Subtangent; but $2hx-xx = \frac{h}{p} \times yy$, and if we put $\frac{h}{p}=e$, then $2hx - x = eyy$, or

$xx-2bx=-eyy$; by adding bb to both Sides of the Equation, $xx-2bx+bb=bb-eyy$, and by Evolution $b-x=\overline{bb-eyy}|^{\frac{1}{2}}$ whence $\frac{2bx-xx}{b-x}=\frac{yye}{\overline{bb-eyy}|^{\frac{1}{2}}}$ the Square of which is $\frac{y^4e^2}{bb-eyy}$, then NM*q* + MT*q*=NT*q*, or $\frac{y^4e}{bb-eyy}+yy=\frac{y^4e^2+y^2b^2-ey^4}{bb-eyy}$ and NT*q*. NM*q* : : QB . HF, or $\frac{y^4e^2+bbyy-ey^4}{bb-eyy}$. yy : : b . $\frac{b^3y-yeby^4}{y^2e^2+bbyy-ey^4}=\frac{b^3-eby^2}{y^2e^2+bb-ey^2}$.

Now, if we call $\frac{c}{2r}yy$ the Baſe of the Solid, the Fluxion of the Baſe may be called $2y\dot{y}$, which multiplied into $\frac{b^3-eby^2}{y^2e^2+bb-ey^2}$ gives $\frac{2b^3y\dot{y}-2eby^3\dot{y}}{y^4e^2+bb-ey^2}$ the Fluxion of the Solid generated by revolving the Curve QHL about the Axis QB, the Solidity of which expreſſes the Reſiſtance of the Spheroid.

Put $e^2-e=m$, then the Fluxion of the Solid will become $\frac{2b^3y\dot{y}-2eby^3\dot{y}}{bb+my^2}=\frac{b^3\times 2y\dot{y}}{bb+my^2}-\frac{eb\times 2y^3\dot{y}}{bb+my^2}$;

To find the Fluent of $\frac{b^3 \times 2y\dot{y}}{bb+my^2} = \frac{\frac{b^3}{m} \times 2y\dot{y}}{\frac{bb}{m}+yy}$, put $\frac{bb}{m}+y^2=z$, then $yy=z-\frac{bb}{m}$ and $2y\dot{y}=\dot{z}$ whence

$\frac{\frac{b^3}{m} \times 2y\dot{y}}{\frac{bb}{m}+yy} = b^3 \times \frac{\dot{z}}{z}$ the Fluent of which is $b^3 \times \text{Log.}\, z$, and reſtoring $\frac{bb}{m}+yy$ for z, it will be $\frac{b^3}{m} \times \text{Log.}\, \overline{\frac{bb}{m}+yy}$, but becauſe when y vaniſhes, the whole Expreſſion ſhould vaniſh, the true Fluent will be

$$\frac{b^3}{m} \times \text{Log.}\, \frac{\frac{bb}{m}+yy}{\frac{bb}{m}}.$$

And to find the Fluent of $-\frac{eb \times 2y^3\dot{y}}{bb+my^2} = -\frac{\frac{eb}{m} \times 2y^3\dot{y}}{\frac{bb}{m}+yy}$, put as before $\frac{bb}{m}+yy=z$, then $yy=z-\frac{bb}{m}$ and $y^4=z^2-2z \times \frac{b^2}{m}+\frac{b^4}{m^2}$, $4y^3\dot{y}=2z\dot{z}-2\dot{z}$

$\times\frac{b^2}{m}$, or $2y^3\dot{y} = z\dot{z} - \dot{z}\times\frac{b^2}{m}$, whence $-\frac{\frac{eb}{m}\times 2y^3\dot{y}}{\frac{bb}{m}+yy}$

$= -\frac{\frac{eb}{m}\times z\dot{z}}{z} + \frac{\frac{eb}{m}\times\frac{b^2}{m}\times\dot{z}}{z}$ the Fluent of the former Term $-\frac{eb}{m}\times\dot{z}$ is $-\frac{eb}{m}\times z = -\frac{eb}{m}\times\overline{\frac{bb}{m}+y^2} = -\frac{eb}{m}\times\frac{bb+my^2}{m}, = -\frac{eb^3}{m^2}-\frac{eby^2}{m}$.

And the Fluent of the latter Term $\frac{\frac{eb}{m}\times\frac{b^2}{m}\times\dot{z}}{z} = \frac{eb^3}{mm}\times\frac{\dot{z}}{z}$ is $\frac{eb^3}{m^2}\times$ Log. z, and by reſtoring $\frac{bb}{m}+yy$ for z, the Fluent will be $\frac{eb^3}{m^2}\times$ Log. $\frac{\frac{bb}{m}+yy}{\frac{bb}{m}}$

The Aggregate of all the Terms of the flowing Quantity is $\frac{b^3}{m}\times$ Log. $\frac{\frac{bb}{m}+yy}{\frac{bb}{m}} + \frac{eb^3}{m^2}\times$ Log. $\frac{\frac{bb}{m}+yy}{\frac{bb}{m}} - \frac{eby^2}{m} - \frac{eb^3}{m^2}$, but becauſe the whole

Expreſſion muſt vaniſh when $y=0$, the true Fluent or Reſiſtance is $\frac{c}{2r}\times\overline{\frac{b^3}{m}+\frac{eb^3}{m^2}\times\text{Log.}\frac{bb+myy}{bb}-\frac{eby^2}{m}}$

for $\dfrac{\frac{bb}{m}+yy}{\frac{bb}{m}}=\dfrac{bb+myy}{bb}$.

If we put as before, $b=66,9$, $y=33\frac{1}{3}$, then $p=16,6085$ nearly, and the Reſiſtance will come out 65603 : the Solidity of the Spheroid is 155683, by which the Reſiſtance being divided gives ,42138.

If the Spheroid degenerates into a Sphere, then $e=1$ and $m=0$, now the Fluxion of $bb+myy$, is $2my\dot{y}\times$ $\overline{bb+myy}|^{-1}$ & $\overline{bb+myy}|^{-1}=\frac{1}{bb}-\frac{myy}{b^4}+\frac{m^2y^4}{b^6}-\frac{m^2y^6}{b^8}$ which multiplied into $2my\dot{y}$ gives $\frac{2my\dot{y}}{bb}-\frac{2m^2y^3\dot{y}}{b^4}+\frac{2m^3y^5\dot{y}}{b^6}-\frac{2m^4y^7\dot{y}}{b^8}$ the Fluent of which is $\frac{my^2}{bb}-\frac{m^2y^4}{2b^4}+\frac{m^3y^6}{3b^5}-\frac{m^4y^8}{4b^8}$ &c. the Log. of $bb+myy$, which multiplied into $\frac{b^3}{m}-\frac{eb^3}{m^2}$ gives $by^2-\frac{my^4}{2b}$

+

$+\frac{m^2y^5}{3b^3}-\frac{m^3y^7}{4b^5}$ &c. $+\frac{eby^2}{m}-\frac{ey^4}{2b}+\frac{mey^6}{3b^3}-\frac{em^2y^8}{4b^5}$, but when $m=0$ it will be $by^2+\frac{eby^2}{m}-\frac{ey^4}{2b}$ becauſe the other Terms will all vaniſh, and ſubtracting $\frac{eby^2}{m}$ it will be $by^2-\frac{ey^4}{2b}$, and when $y=b$ and $e=1$, we ſhall have $\frac{c}{2r}\times\frac{b^3}{2}$ the Reſiſtance or half the circumſcribed Cylinder.

PROP. VII.

To Inveſtigate the Reſiſtance of an Hyperbolical Conoid.

LET QENB, *Fig.* 14, be the generating Hyperbola, $QB=b$ the Axis of the given Solid: and put the Tranſverſe Diameter of the Hyperbola $=2d$, the Semiparameter $=p$, and let y denote an Ordinate and x the Abſciſſa proper to it.

By the Property of the Hyperbola $yy=\frac{p}{d}\times\overline{2dx+x^2}$ by Evolution $y=\frac{p^{\frac{1}{2}}}{d^{\frac{1}{2}}}\times\overline{2dx+xx}|^{\frac{1}{2}}$ the Fluxion of which of which is $\dot{y} = \frac{p^{\frac{1}{2}}}{d^{\frac{1}{2}}} \times \frac{d\dot{x}+x\dot{x}}{\overline{2dx+xx}|^{\frac{1}{2}}}$ then $nr \,.\, rN ::$ NM . MT, or ſubſtituting their Values, $\frac{p^{\frac{1}{2}}}{d^{\frac{1}{2}}} \times \frac{d\dot{x}+x\dot{x}}{\overline{2dx+xx}|^{\frac{1}{2}}} \,.\, \dot{x} :: \frac{p^{\frac{1}{2}}}{d^{\frac{1}{2}}} \times \overline{2dx+xx}|^{\frac{1}{2}} \,.\, \frac{2dx\dot{x}+x^2\dot{x}}{d\dot{x}+x\dot{x}} = \frac{2dx+xx}{d+x}$ the Subtangent. And if we put $\frac{d}{p}=e$, then $yye=2dx+xx$ and $\frac{2dx+xx}{d+x}=\frac{yye}{d+x}$, by adding dd to both Sides of the Equation $dd+2dx+xx=dd+yye$, and by Evolution $d+x=\overline{dd+yye}|^{\frac{1}{2}}$ therefore $\frac{2dx+xx}{d+x}=\frac{yye}{\overline{dd+yye}|^{\frac{1}{2}}}$, the Square of which is $\frac{y^4e^2}{dd+yye}$. and $\frac{y^4e^2}{dd+yye}+yy=\frac{y^4e^2+d^2y^2+ey^4}{dd+yye}$, the Square of the Tangent: Now NTq . NMq :: QB . FH, or ſubſtituting their Values, $\frac{y^4e^2+d^2y^2ey^4}{dd+yye} \,.\, yy :: h \,.\, \frac{hd^2y^2+hey^4}{y^4e^2+d^2y^2+ey^4}=\frac{hd^2+hey^2}{y^2e^2+d^2+ey^2}$, and putting $e^2+e=m$ it will be $\frac{hd^2+hey^2}{d^2+my^2}$.

If we put $\frac{c}{2r} \times yy$ for the Baſe of the Solid; because $\frac{c}{2r}$ is a ſtanding Expreſſion, ſhewing the Ratio of a Circle to a Square whoſe Side is the Radius of the Circle: the Fluxion of the Baſe may be called $2y\dot{y}$ which multiplied into $\frac{hd^2+hey^2}{d^2+my^2}$ will give $\frac{hd^2 \times 2y\dot{y}}{d^2+my^2}$ $+ \frac{2hey^3\dot{y}}{d^2+my^2}$, the Fluxion of the Solid whoſe Solidity expreſſes the Reſiſtance of the given Solid.

In Order to find which, put $d^2+my^2=z$, then $y^2 = \frac{z-d^2}{m}$, $2y\dot{y} = \frac{\dot{z}}{m}$, $y^4 = \frac{zz-2d^2z+d^4}{m^2}$, $4y^3\dot{y} = \frac{2z\dot{z}-2d^2\dot{z}}{m^2}$ and $2y^3\dot{y} = \frac{z\dot{z}-d^2\dot{z}}{m^2}$, by ſubſtituting which in the Fluxion of the Solid, we ſhall inſtead of the firſt Term $\frac{hd^2 \times 2y\dot{y}}{d^2+my^2}$ have $\frac{hd^2 \times \frac{\dot{z}}{m}}{z} = \frac{hd^2}{m} \times \frac{\dot{z}}{z}$ the Fluent of which is $\frac{hd^2}{m} \times$ Log. z, and reſtoring d^2+myy for z we ſhall have $\frac{hd^2}{m}\times\text{Log.}\,\overline{\frac{d^2+myy}{d^2}}$ becauſe the Log. of z muſt vaniſh when $y=0$.

And inſtead of the ſecond Term $\frac{he\times 2y^3\dot{y}}{d^2+my^2}$, there will ariſe $\frac{hez\dot{z}}{m^2z}-\frac{hed^2\dot{z}}{m^2z}$, the Fluent of which is $\frac{hez}{m^2}-\frac{hed^2}{m^2}\times\text{Log.}\,z$, and reſtoring d^2+myy for z, we ſhall have $\frac{hed^2}{m^2}+\frac{heyy}{m}-\frac{hed^2}{m^2}\times\text{Log.}\,\overline{\frac{d^2+myy}{d^2}}$. The Aggregate of all which is $\overline{\frac{hd^2}{m}-\frac{hed^2}{m^2}}\times\text{Log.}\,\overline{\frac{a^2+myy}{d^2}}+\frac{heyy}{m}+\frac{hed^2}{m}$.

But, becauſe when y vaniſhes the whole Expreſſion muſt vaniſh with it, the true Fluent will be $\overline{\frac{hd^2}{m}-\frac{hea^2}{m^2}}\times\text{Log.}\,\overline{\frac{d^2+myy}{d^2}}+hey^2$ which Expreſſion multiplied into $\frac{c}{2r}$ gives the Reſiſtance required.

If $h=d$, or the Altitude be equal to half the tranſverſe Diameter as in the Spheroid it will then be

$$\overline{\frac{h^3}{m} - \frac{eh^3}{m^2}} \times \text{Log.}\ \overline{\frac{hh+myy}{hh}} + \frac{hey^2}{m}.$$

Now, if we put $h=d=66,9$, and $y=33\frac{1}{3}$, then $p=5,5361$: And the Reſiſtance will come out 38058: The Solidity of this Hyperbolical Conoid is 103789, by which the Reſiſtance being divided will give ,36668.

PROP. VIII.

To Inveſtigate the Fruſtum of a Cone, *which ſhall meet with leſs Reſiſtance, than any other Fruſtum of the ſame Baſe and Altitude.*

PUT $OC=r$, *Fig.* 15. $OD=h$, and $OS=x$: Then $CSq=xx+rr$, and $OS\ (x)\ .\ OC\ (r)\ ::\ DS\ (x-h)\ .\ DF=\frac{rx-rh}{x}$. Again, if $CSq\ (xx+rr)\ .\ OCq\ (rr)\ ::\ OD\ (h)\ .\ AE=\frac{rrh}{xx+rr}$.

Then the Reſiſtance of this Fruſtum generated by revolving OCFD about the Axis OD may be expreſſed by the Solidity generated in turning the *Fig.* OIHAED about the ſaid Axis OD.

The Content of the Cylinder generated by the Parallelogram AEDG in this Rotation about OD may be repreſented by $\frac{r^4b}{xx+rr}$, and that of IHGO in the ſame Rotation by $\frac{br^2x^4-2r^2x^3b^2+b^3r^2x^2}{x^4+r^2x^2}$: the whole Reſiſtance of the Fruſtum will therefore be $\frac{r^4b}{xx+rr}+\frac{br^2x^4-2r^2x^3b^2+b^3r^2x^2}{x^4+r^2x^2}=$ $\frac{r^4bx^2+br^2x^4-2r^2x^3b^2+b^3r^2x^2}{x^4+r^2x^2}$ equal to $\frac{r^4b+br^2x^2-2r^2xb^2+b^3r^2}{xx+rr}$, which by the Nature of the Queſtion is to be a Minimum.

The Fluxion of the Numerator $r^4b+br^2x^2-2r^2xb^2+b^3r^2$ is $2br^2x\dot{x}-2b^2r^2\dot{x}$, which multiplied into the Denominator x^2+r^2 gives $2br^2x^3\dot{x}-2b^2r^2x^2\dot{x}+2br^4x\dot{x}-2b^2r^4\dot{x}$.

And the Fluxion of the Denominator x^2+r^2 is $2x\dot{x}$, which multiplied into the Numerator $r^4b+br^2x^2-2r^2b^2x+b^3r^2$, will produce $2br^4x\dot{x}+2br^2x^3\dot{x}-4b^2r^2x^2\dot{x}+2b^3r^2x\dot{x}$; the Fluxion therefore of the Reſiſtance $\frac{r^4b+br^2x^2-2r^2xb^2+b^3r^2}{x^2+r^2}$, is $\frac{2b^2r^2x^2\dot{x}-2b^2r^4\dot{x}-2b^3r^2x\dot{x}}{\overline{x^2+r^2}|^4}=0$, if a Minimum,

whence $2b^2r^2x^2\dot{x}-2b^2r^4\dot{x}-2b^3r^2x\dot{x}=0$, and dividing by $2b^2r^2\dot{x}$, we ſhall have $x^2-r^2-bx=0$, or $x^2-bx=r^2$, and adding $\frac{1}{4}bb$, or the Square of half the known Coefficient b, to both Sides of the Equation, it will be $x^2-bx+\frac{1}{4}bb=r^2+\frac{1}{4}bb$, and by Evolution $x-\frac{1}{2}b=\overline{r^2+\frac{1}{4}bb}|^{\frac{1}{2}}$, now $QS=x-\frac{1}{2}b$, and $CQ=\overline{r^2+\frac{1}{4}bb}|^{\frac{1}{2}}$ whence follows that eaſy Conſtruction, given by Sir *Iſaac Newton*;

Biſect the Altitude OD in Q, and produce OQ to S, ſo that QS may be equal to QC, and S will be the Vertex of the Cone whoſe Fruſtum is ſought.

Now, if $b=66{,}9$, and $r=33\frac{1}{3}$, as in the Solids whoſe Reſiſtances we have compared before, then OS=80,673 FD=5,6909 AE=11,4216, and the Reſiſtance of the Fruſtum will be 41045, and its Solid Content 93435, by which the Reſiſtance being divided will give 43929.

The Reſiſtance of the inſcribed Cone generated by revolving the Triangle OCD about the Axis OD is repreſented by the Solidity of the Cylinder generated by the Parallelogram KEDL in the Same Rotation, in which the Altitude KE is equal to $\frac{OD\times OCq}{CDq}=13{,}3053$, whence the Reſiſtance of the Cone will be 46444: its Solidity is one Third of the circumſcribed Cylinder, or 77841, by which the Reſiſtance being divided will give ,52729.

And the Reſiſtance of this Cone, to the Reſiſtance of that Fruſtum of a Cone of the ſame Baſe and Altitude, is as 46444 to 41045, and its Solidity to the Solidity of the Fruſtum no more than as 77841 to 93435.

And from this Conſtruction of the Fruſtum of the Cone of the leaſt Reſiſtance (as Sir *Iſaac Newton* obſerves) it follows: That

If a Solid be generated by revolving any Curve of an Elliptical or Oval-like Figure DNFB, *Fig.* 16. about its Axis CB: and another Solid be generated by revolving DNFGB about the ſame Axis, in which GB is Perpendicular to the Axis, and FG be a Tangent to the Curve in the Point H, making the Angle FGB 135 Degrees, the Reſiſtance of the latter Solid will be leſs than that of the former: And this will appear from the Conſideration of the foregoing Conſtruction of the Fruſtum of the Cone which meets with the leaſt Reſiſtance: For, if QB were infinitely ſmall, it is plain that $FQ = QP$, or the Angle $FPQ = 45$, or Angle $FGB = 135$ Degrees.

Let the Curve DNFB be an Ellipſis: Then (as was obſerved before) the Reſiſtance of the Spheroid may be expreſſed by the Content of the Solid generated by the Rotation of the Curve LMC about the ſame Axis; and that of DNFGB by the Content of the Solid generated by LMEO in the ſame Rotation: where univerſally $ME = QF - GB$, and $GE = EO = \frac{1}{2} BC$, becauſe, $FPq = 2FQq$, and by the foregoing Method of comparing the Reſiſtances of the Solids it will be $2FQq \,.\, FQq :: CB \,.\, \frac{1}{2}CB = MS$.

Let as before, h denote the Altitude CB, p the Semiparameter, y an Ordinate, and put $\frac{h}{p}=e$.

The Subtangent will be $\frac{y^2e}{\overline{hh-eyy}|^{\frac{1}{2}}}$ as was ſhewn before, in the Inveſtigation of the Reſiſtance of the Spheroid; and in this Caſe $\frac{y^4e^2}{hh-eyy}=yy$, and dividing by yy we ſhall have $\frac{y^2e^2}{hh-eyy}=1$, whence $y^2e^2=hh-eyy$, or $y^2e^2+y^2e=hh$, and dividing by e^2+e, $y^2=\frac{h^2}{e^2+e}$, but $e^2+e=\frac{h^2}{p^2}+\frac{h}{p}$ equal to $\frac{h^2+ph}{p^2}$ whence $\frac{h^2}{e^2+e}=\frac{p^2h^2}{h^2+ph}=\frac{p^2h}{h+p}=y^2$ as above, and by Evolution $\frac{ph^{\frac{1}{2}}}{\overline{h+p}|^{\frac{1}{2}}}=y=\text{QF}=\text{QP}$.

And to find the Value of QB $=x$; $\frac{h^2}{e^2+e}=$ $\frac{2hx-xx}{e}$ or $\frac{h^2}{e+1}=2hx+xx$, or which is the ſame, $xx-2hx=-\frac{h^2}{e+1}$, and by compleating the Square, $xx-2hx+xx=hh-\frac{h^2}{e+1}$; by Evolution

$b-x=\overline{bb-\frac{b^2}{e+1}}\Big|^{\frac{1}{2}}$ or $b-\overline{bb-\frac{b^2}{e+1}}\Big|^{\frac{1}{2}}=x$: But

$bb-\frac{b^2}{e+1}=\frac{bbe}{e+1}=\frac{bb\times\frac{b}{p}}{\frac{b}{p}+1}$, now $bb\times\frac{b}{p}=$ $\frac{b^3}{p}$ and $\frac{b}{p}+1=\frac{b+p}{p}$, therefore $\frac{bbe}{e+1}=\frac{b^3}{b+p}$, whence $b-\overline{bb-\frac{b^2}{e+1}}\Big|^{\frac{1}{2}}=b-\overline{\frac{b^3}{b+p}}\Big|^{\frac{1}{2}}=x=$ QB the Abſciſſa.

Now, if we put $b=66,9$, and $p=16,6085$ as before, then QF $=$ 14,8654, GB $=$ 8,8775, and QB 5,9879: by the Help of which the Reſiſtance of the Fruſtum of the Cone generated by FGBQ in its Rotation about its Axis will be found to be 31527.

And the Reſiſtance of the Solid generated by FBQ in the ſame Rotation will be found 32989, which ſubtracted from 65603 the Reſiſtance of the whole Spheroid, leaves 32614, to which if we add 31527 we ſhall have 64141, the Reſiſtance of the Solid generated by revolving the Figure DNFGBC about the ſame Axis CB: which is to the Reſiſtance of the Spheroid, as 64141 to 65603, or as 64 to 65½, nearly.

Let the given Curve DNFB, *Fig.* 17. be a Parabola, put the Semiparameter $=a$, and the Altitude CB$=b$ as before: then, if the Angle FPQ be an Angle of 45 Degrees, FQ=QP$=a$, and BP=QB=GH $=\frac{1}{2}$ QF, whence the Reſiſtance of the Fruſtum of the Cone generated by the Rotation of FGBQ about its Axis, as repreſented by the Solid generated by the Rotation of BSMEOC about the ſame Axis, may be expreſſed by $c \times \frac{1}{2} b \times yy \times \frac{yy}{4}$ (c ſtanding for the Ratio of the Periphery of a Circle to its Diameter) $= c \times \frac{1}{2} b \times \frac{5yy}{4} = c \times \frac{5}{8} byy = c \times \frac{5}{8} baa$, or $\frac{5}{8}$ the Reſiſtance of a Cylinder whoſe Radius of the Baſe is SB, or the Semiparameter.

And if we put CB=66,9, CD=$33\frac{1}{3}$, as before, then $y = a =$ FQ $= 8,3042$; $yy = aa = 68,9608$, and the Reſiſtance of the Fruſtum of the Cone generated by FGBQ in its Rotation about its Axis will come out 9058 : And the Reſiſtance of FBQ in the ſame Rotation will be found 10044 : the Reſiſtance of the whole Paraboloid generated by CDNFB was ſhewn before, to be 41169, from which if we deduct 10044, there will remain 31125, to which if we add 9058, we ſhall have 40183, the Reſiſtance of the Solid generated by revolving the Figure CDNFGB, about the ſame Axis CB, which is to the Reſiſtance of the Paraboloid, as 40183 to 41169, or 40 to 41 nearly. And this Propoſition, Sir *Iſaac Newton* ſays, he ſuppoſes may be uſeful in *Ship-Building* : but, with this Remark, that

If the Figure DNFG, *Fig.* 18. be a Curve of ſuch a Kind : if from any Point N, a Perpendicular NM be let fall on the Axis CB, and from a given Point G be drawn a Right Line GR, parallel to a Right Line touching the Figure in N, and cutting the Axis produced in R : MN will be to GR, as GR cubed to 4BR × GB*q*: the Solid generated by revolving this Figure about its Axis CB will meet with leſs Reſiſtance than any other Circular Solid of the ſame Baſe and Altitude: and here it naturally ſollows,

PROP. IX.

To Inveſtigate the Property of a Curve, *which revolved about its Axis, generates a* Solid *of the leaſt Reſiſtance.*

IT was obſerved in the foregoing Propoſition, that when the Altitude of the Fruſtum of a Cone of the leaſt Reſiſtance becomes infinitely ſhort, the Side of the Cone will make an Angle with the Axis of 45 Degrees, and conſequently a Tangent applied in the Head of the leaſt Ordinate of the Curve ſought will cut the Axis produced in the ſame Angle.

Let CD, *Fig.* 19. be the Axis of the Curve, IE a Right Line Parallel to the Axis, and the Points K and D given; draw the Ordinates KC, NM and FD infinitely near to each other, put the Ordinate NM $=y$, and KI $=d$ invariable, and let the Variable Quantities be CM$=x$, HF$=e$, and the leaſt Ordinate DF$=n$.

Now, if the Force of the Fluid ſtriking directly on IK at the Diſtance IC or NM be called dy, then $d^2+x^2 \,.\, d^2 :: dy \,.\, \frac{d^2y}{d^2+x^2}$, the Reſiſtance on the Surface generated by the infinitely little Part of the Curve KN in its Rotation about its Axis.

And becauſe HN=HF; $2ee \,.\, ee :: ne \,.\, \frac{ne}{2}$, the Reſiſtance generated by NF, and the Reſiſtance of DF will be $\frac{nn}{2}$; the Reſiſtance therefore of the contiguous Particles of the Solid generated by KNFD in its Rotation about its Axis may be repreſented by $\frac{d^3y}{d^2+x^2}+\frac{ne+nn}{2}$, which is to be a Minimum, its Fluxion therefore is $-\frac{2d^3y\dot{x}}{\overline{d^2+x^2}|^2}+\frac{\dot{n}e+n\dot{e}-2n\dot{n}}{2}=0$, but CD is a conſtant Quantity, therefore $-\dot{x}=\dot{e}$; and CD $-x=$ NM $-n$, therefore $\dot{x}=\dot{n}$, whence $\frac{\dot{n}e+n\dot{e}+2n\dot{n}}{2}=\frac{\dot{x}e-n\dot{x}+2n\dot{x}}{2}=\frac{\dot{x}e+n\dot{x}}{2}$, and then the Fluxion of the Reſiſtance will be $-\frac{2d^3yx\dot{x}}{\overline{d^2+x^2}|^2}+\frac{\dot{x}e+n\dot{x}}{2}=0$, and dividing by $\dot{x}$; $\frac{2d^3yx}{\overline{d^2+x^2}|^2}+\frac{e+n}{2}$, Or $\frac{4d^3yx}{\overline{d^2+x^2}|^2}=e+n$, and becauſe e is infinitely ſmall in reſpect of n, $\frac{4d^3yx}{\overline{d^2+x^2}|^2}=n$, $4d\ yx=\overline{d^2+x^2}|^2+n$, or $4d^3x\ .\ \overline{d^2+x^2}|^2\ ::\ n\ .\ y$, but x is the Fluxion of the Axis, and d the Fluxion of the Ordinate : therefore if GB$=a$, and BR$=z$, be the Legs of a Triangle ſimilar to the Fluxionary Triangle IKN, it will be $4a^3z\ .\ a^4+2a^2z^2+z^4\ ::\ n\ .\ y$, and y equal to $\frac{a^4+2aazz+z^4}{4aaz}\times\frac{n}{a}$, whereby it is plain that Sir *Iſaac Newton*'s Property of this Curve makes the leaſt Ordinate $n=a$ the Parameter, and then it will be, MN to GR as GR cubed, to 4BR×GB ſquared.

And tho' a Curve deſcribed by this Property will approach nearer to the Axis, than the Point where a Tangent makes an Angle with the Axis of 45 Degrees, and recedes from it again, becoming then convex towards the Axis, yet after it has paſſed that Point it will not generate a Solid of the leaſt Reſiſtance ; becauſe it has loſt one of its chief Properties.

Beſides, if DNGB, *Fig.* 20. were the Curve, then DNFB was ſhewn to reſiſt leſs than DNGB, and the Fruſtum of the Cone generated by NFB in its Rotation about its Axis MB, would reſiſt leſs than the Fruſtum generated by revolving N H B about the ſame Axis in which MC=CB and CS=CN, which is abſurd : becauſe, this was ſhewn to reſiſt the leaſt of all Fruſtums of the ſame Baſe and Altitude.

If we put x for an Abſciſſa of the Curve, its Fluxion $=\dot{x}$, y an Ordinate, its Fluxion $\dot{y}$, GB$=a$, and BR$=z$, as before, then in the ſimilar Triangles IKN and BGR, it will be GB . IK : : BR . IN, or

$a \,.\, \dot{y} :: z \,.\, \dot{x} = \frac{z\dot{y}}{a}$ but $y = \frac{a^4+2aazz+z^4}{4aaz} = \frac{a^2}{4z} + \frac{z}{2} + \frac{z^3}{4aa}$, the Fluxion of which is $\dot{y} = \frac{3z^2\dot{z}}{4aa} + \frac{\dot{z}}{2} - \frac{aa\dot{z}}{4zz}$, whence $\dot{x}$ or $\frac{z\dot{y}}{a} = \frac{3z^3\dot{z}}{4a^3} + \frac{z\dot{z}}{2a} - \frac{a\dot{z}}{4z}$ the flowing Quantity of which is $x = \frac{3z^4}{16a^3} + \frac{z^2}{4a} - \frac{a}{4} \times$ Log. z, but when $x=0$ then $z=a$, which being ſubſtituted for z, we ſhall have $\frac{3a^4}{16a^3} + \frac{a^2}{4a} - \frac{a}{4} \times$ Log. $a = \frac{7}{16}a + \frac{1}{4}a \times$ Log. a, which being ſubtracted from $\frac{3z^4}{16a^3} + \frac{zz}{4a} - \frac{a}{4} \times$ Log. z, leaves $\frac{3z^4}{16a^3} + \frac{zz}{4a} - \frac{7a}{16} - \frac{a}{4} \times$ Log. $\frac{z}{a}$, the Abſciſſa required.

And putting $GB=a=1$, it will be $4z \,.\, 1+z^2 :: 1+z^2 \,.\, y$ the Ordinate: and $16 \,.\, 3z^3+4 :: z^2 \,.\, 4th$, from which fourth Term ſubtracting $\frac{7}{16} + \frac{1}{4}$ Log. z

leaves the Abſciſſa proper to the Ordinate found as above.

PROP. X.

To Inveſtigate the Reſiſtance of a Solid *of the leaſt Reſiſtance turned about its Axis.*

LET QENGB, *Fig.* 21. be the generating Figure, draw KF parallel to the Axis QB, meeting with the Curve in N, draw NM perpendicular to the Axis, and NI perpendicular to the Curve, and the Force with which a Particle of the Fluid ſtriking directly on the Perpendicular Head of the Cylinder in the Point F, is to the Force with which it would ſtrike obliquely on the given Solid in the Point N, as NIq to NMq, or becauſe GR is parallel to a Tangent applied in the Point N; as GRq to GBq, and putting BR$=z$, GB$=a$, and conſequently GR$=\overline{zz+aa}|^{\frac{1}{2}}$, and taking any determinate Quantity as the Altitude QB$=b$, as before, it will be GRq

$$. \mathrm{GB}q :: \mathrm{QB} . \mathrm{HF}, \text{ or } zz+aa . aa :: b . \frac{aab}{zz+aa}$$

and if this be done in every Point of the given Curve, the Point H will deſcribe the Curve OHD, and the Reſiſtance of the given Solid will be to the Reſiſtance of the circumſcribed Cylinder, as the Content of the Solid generated by revolving the Figure QOHDLB about the Axis QB, is to the Content of the Circumſcribed Cylinder.

Put as before HF$= \frac{aah}{zz+aa}$, then KH$=h - \frac{aah}{zz+aa} = \frac{zzh}{zz+aa}$, the Fluxion of which is $\frac{2aahz\dot{z}}{\overline{zz+aa}|^2}$ and put the Baſe $= \frac{c}{2r} \times yy$, but by the Property of the Curve $y = \frac{z^4+2aazz+a^4}{4aaz}$, whence $\frac{c}{2r} \times yy = \frac{c}{2r} \times \frac{\overline{z^4+2aazz+a^4}|^2}{16a^4zz}$ which multiplied into $\frac{2aahz\dot{z}}{\overline{zz+aa}|^2}$ gives $\frac{c}{2r} \times \frac{2aahz\dot{z}}{\overline{zz+aa}|^2} \times \frac{\overline{zz+aa}|^4}{16a^4zz}$ or becauſe $\frac{c}{2r} \times h$ is conſtant, it may be omitted, and then it will be $2aaz\dot{z} \times \frac{\overline{zz+aa}|^2}{16a^4zz} = \frac{\dot{z}}{8aaz}$ multiplied by $\overline{z^4+2aazz+a^4} = \frac{z^3\dot{z}}{8aa} + \frac{z\dot{z}}{4} + \frac{aa\dot{z}}{8z}$, the Fluxion of the Solid; the flowing Quantity of which is $\frac{z^4}{32aa} + \frac{z^2}{8} + \frac{aa}{8} \times \text{Log.}\frac{z}{a}$ to which if we add the Cylinder generated by the Parallelogram ADBL, in the ſame Rotation, which in this Caſe is $\frac{\overline{zz+aa}|^4}{16a^4z^2} \times \frac{aa}{zz+aa}$ or $\frac{\overline{zz+aa}|^3}{46aazz} = \frac{z^6+3a^2z^4+3a^4z^2+a^5}{16a^2z^2} = \frac{z^4}{16a^2} + \frac{3z^2}{16} + \frac{3a^2}{16} + \frac{a^4}{16z^2}$; we ſhall have $\frac{3z^4}{32aa} + \frac{5z^2}{16} + \frac{3a^2}{16} + \frac{a^4}{16z^2} + \frac{aa}{8} \times \text{Log.}\ \frac{z}{a}$, but when $z=a$, it will be $\frac{3a^2}{32} + \frac{5a^2}{16} + \frac{3a^2}{16} + \frac{a^2}{16} + \frac{aa}{8} \times \text{Log.}\ 1$, or $\frac{21a^2}{32}$ which ſhould be a^2, but is deficient $\frac{11}{32}a^2$, therefore $\frac{3z^4}{32aa} + \frac{5z^2}{16} + \frac{17a^2}{32} + \frac{a^4}{16z} + \frac{aa}{8} \times \text{Log.}\ \frac{z}{a}$ is the true Fluent, which multiplied into $\frac{c}{2r} \times b$, gives the Reſiſtance enquired after.

And putting $b=66{,}9$ and $=33\frac{1}{3}$, then $z=12$, and the Reſiſtance of this Solid will be 37265, the Solidity of the circumſcribed Cylinder being put for its Reſiſtance.

PROP.

PROP. XI.

To Cube the Solid *of the leaſt Reſiſtance.*

PUT the Radius of the Baſe NM=y, *Fig*. 22. its Diameter $2y$, and let c be a ſtanding Expreſſion, ſhewing the Proportion of the Area of a Circle to its circumſcribed Square: the Area of the Baſe will then be $c\times 4yy$, but becauſe c is a ſtanding Quantity we may omit it, and call the Baſe $4yy$, then if the Altitude MB of the Solid be x; its Fluxion $\dot{x}$ multiplied into the Baſe will give $4y^2\dot{x}$ the Fluxion of the Solid.

But by the Property of the Curve, putting BR=z, $y=\frac{z^4+2aazz+a^4}{4aaz}$, whence $4yy$ will be equal to

$\frac{z^8+4aaz^6+6a^4z^4+4a^6zz+a^8}{4a^4z^2}$, and $\dot{x}$ equal to

$\frac{3z^4\dot{z}+2az^2\dot{z}-a^4\dot{z}}{4a^3z}$, whence $4yy\dot{x}$ will become

$$\frac{3z^{12}\dot{z}+14a^2z^{10}\dot{z}+25a^4z^8\dot{z}+20a^6z^6\dot{z}+5a^8z^4\dot{z}-2a^{10}z^2\dot{z}-a^{12}\dot{z}}{16a^7z^3}$$

the Fluxion of the Solid; by Reduction equal to

$\frac{3z^9\dot{z}}{16a^7}+\frac{7z^7\dot{z}}{8a^5}+\frac{25z^5\dot{z}}{16a^3}+\frac{5z^3\dot{z}}{4a}+\frac{5az\dot{z}}{16}+\frac{a^3\dot{z}}{8z}-$

$a^5 z$

$\frac{a^5\dot{z}}{16z^3}$ the Fluent of which is $\frac{3a^{10}}{160a^7} + \frac{7z^8}{64a^5} + \frac{25z^6}{96a^3} + \frac{5z^4}{16a} + \frac{5az^2}{32} - \frac{a^3}{8} \times \text{Log.}\,\frac{z}{a} + \frac{a^5}{32z^2}$, Now, when $z=a$, the whole, which ſhould vaniſh, will be $\frac{3a^3}{160} + \frac{7a^3}{64} + \frac{25a^3}{96} + \frac{5a^3}{16} + \frac{5a^3}{32} + \frac{a^3}{32}$, or $\frac{3a^3}{160} + \frac{7a^3}{64} + \frac{25a^3}{96} + \frac{30a^3}{96} + \frac{15a^3}{96} + \frac{3a^3}{96} = \frac{3a^3}{160} + \frac{7a^3}{64} + \frac{73a^3}{96}, = \frac{853}{960} \times a^3$, which being ſubtracted from the flowing Quantity found as above, there will remain $\frac{3z^{10}}{160a^7} + \frac{7z^8}{64a^5} + \frac{25z^6}{96a^3} + \frac{5z^4}{16a} + \frac{5az^2}{32} + \frac{a^5}{32z^2} - \frac{853a^3}{960} - \frac{a^3}{8} \times \text{Log.}\,\frac{z}{a}$, the true Fluent, which multiplied into c will give the Content of the Solid.

And putting $h=66{,}9$ and $y=33\frac{1}{3}$ as before, then $z=12$, and the Content of this Solid will be 106360, by which the Reſiſtance 37265 being divided, we ſhall have ,350376.

PROP. XII.

The Altitude and Baſe of a Cone *being given, to deſcribe a* Trapezium, *whoſe Altitude and Solidity generated by its Rotation about its Axis, ſhall remain the ſame as in the* Cone, *and its Reſiſtance be the leaſt of all* Solids *of the ſame Altitude and Solidity, generated by any* Trapezium *whatever in its Rotation about the ſame Axis.*

LET the right angled Triangle ABC, *Fig.* 23. repreſent the given Cone, and ABFD the Trapezium, put the Axis AB=b, AC=p, AD=y, and EB=x.

If the Solidity of the Cone generated by revolving the Triangle CAB about the Axis AB be called $\frac{ppb}{3}$: The Content of the Solid generated by revolving the Figure ABFD about the ſame Axis will be $yyb-yyx+\frac{yyx}{3}$, and if this be equal to the Solidity of the Cone, then $\frac{ppb}{3}=yyb-yyx+\frac{yyx}{3}$ and

$ppb = 3yyb - 3yyx + yyx$, or $\frac{3yyb - ppb}{2yy} = x$; and

$\frac{ppb}{3b-2x} = y^2$, whence $\frac{pb^{\frac{1}{2}}}{\overline{3b-2x}|^{\frac{1}{2}}} = y$. Now, $bb+pp$

$. pp :: b . \frac{bpp}{bb+pp} =$ GB, and then $\frac{bp^4}{bb+pp}$ will

ſtand for the Content of the Cylinder generated by the Parallelogram GBLK in its Rotation about its Axis, or the Reſiſtance of the given Cone; and in the ſame

Manner $xx+yy . yy :: b . \frac{byy}{xx+yy} =$ BH, or the

Reſiſtance of the Solid enquired after will be repreſented by the Solidity of the Cylinder generated by the Rotation of HBMI about the ſame Axis, which

in this Caſe will be $\frac{by^4}{xx+yy}$, but $y^2 = \frac{p^2b}{3b-2x}$, y^4

$= \frac{p^4b^2}{\overline{3b-2b}|^2} = \frac{p^4b^2}{9bb-12bx+4xx}$ and $xx+yy = xx+$

$\frac{p^2b}{3b-2x} = \frac{3bxx-2x^3+p^2b}{3b-2x}$, therefore $\frac{by^4}{xx+yy}$ will

be equal to the Quotient of $\frac{p^4b^3}{\overline{3b-2x}|^2}$ divided by

$\frac{3bxx-2x^3+p^2b}{3b-2x}$, equal to $\frac{p^4b^3}{3bxx-2x^3+bp^2 \times \overline{3b-2x}}$

$= \frac{p^4b^3}{9bbx^2-12bx^3+3p^2b^2+4x^4-2p^2bx}$ the Fluxion

of which, if a Minimum, will be $-18b^5p^4\dot{x}x +$

Fig. 13.

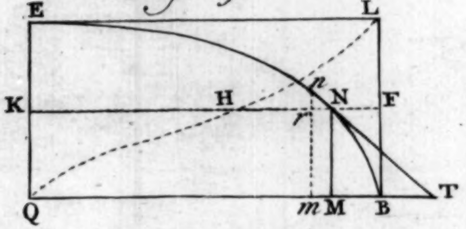


Fig. 14.

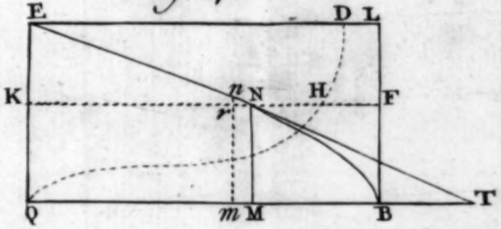


Fig. 15.

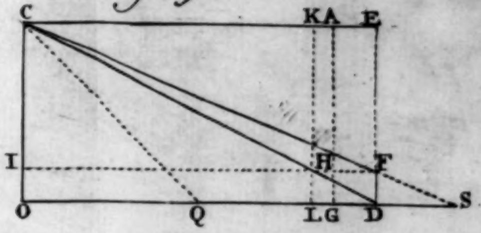


Fig. 16.

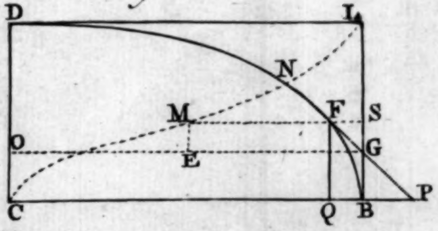


Fig. 17.

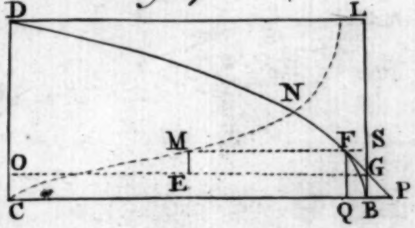


Fig. 18.

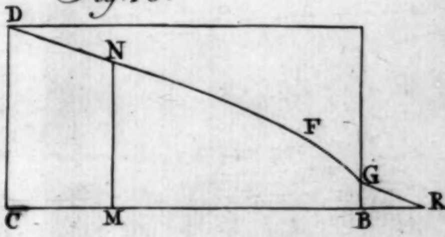


Fig. 19.

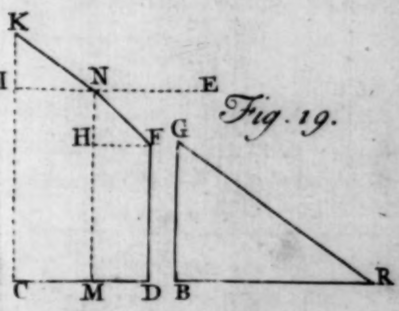


Fig. 20.

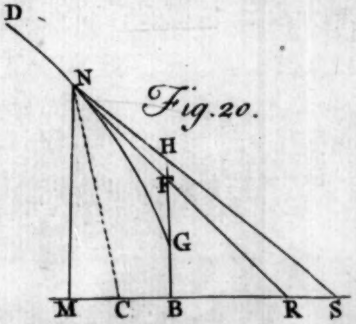


Fig. 21.

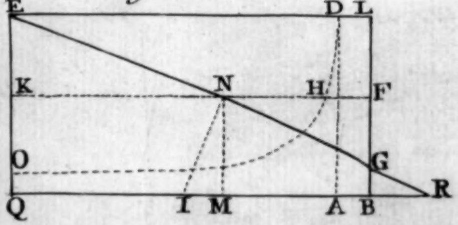


Fig. 22.

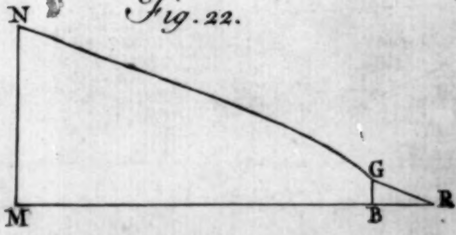
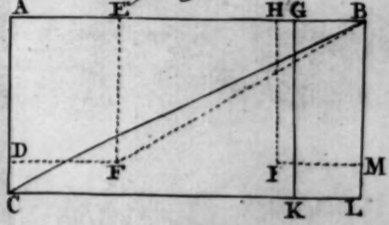


Fig. 23.



J. Mynde sc.

$6b^4p^4x^2\dot{x}-16b^3p^4x^3\dot{x}+2b^4p^6\dot{x}=0$, and dividing by $2b^3p^4\dot{x}$ it will be $-9b^2x+18bx^2-8x+p^2b=0$, or $8x^3-18bx^2+9b^2x=p^2b$, and dividing by 8, $x^3-\frac{9}{4}bx^2+\frac{9}{8}b^2x=\frac{p^2b}{8}$, which Cubic Equation being ſolved will give the Value of x. 8 1150

And if we put AB$=b=66,9$, and AC$=p=33\frac{1}{3}$, as before, then EB$=x=46,1$, EF$=y=26,17$, and HB$=16,308$: So that the Reſiſtance of the Cone being 46444, as was ſhewn before; the Reſiſtance of the Solid generated by revolving this Trapezium ABFD about the Axis AB will be 35101 nearly. And this may ſerve as a general Hint, that the Reſiſtances of Solids depend much upon their greateſt tranſverſe Sections, and it is probable from this, ſome uſeful Obſervations may be made.

F I N I S.